Signal and System

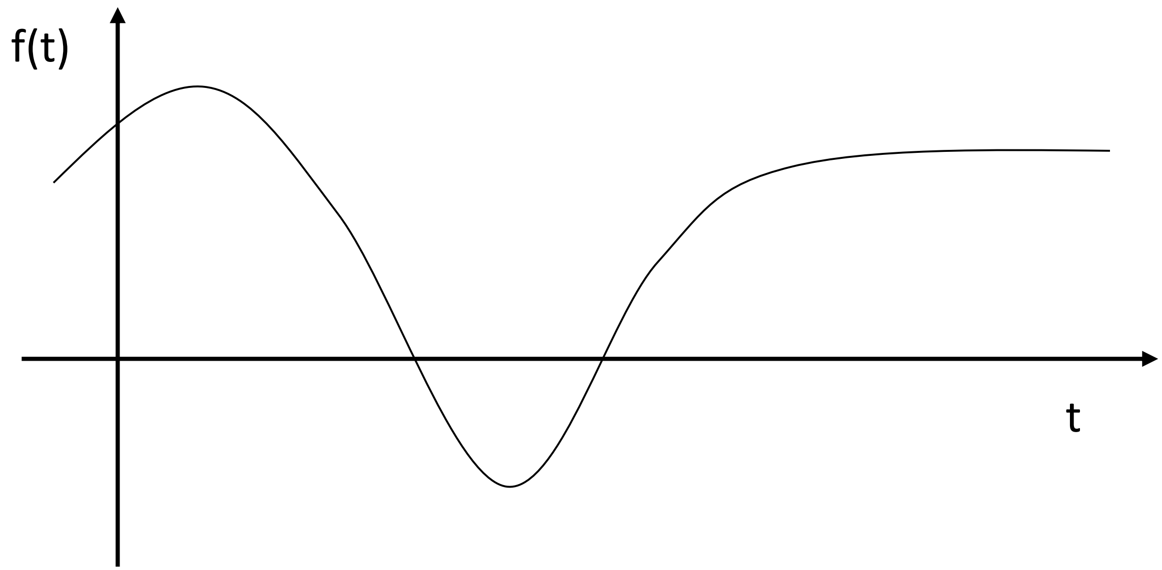
What is a Signal?

- A signal is a pattern of variation of some form
- Signals are variables that carry information

- Examples of signal include:
- Electrical signals
  - Voltages and currents in a circuit
- Acoustic signals
  - Acoustic pressure (sound) over time
- Mechanical signals
  - Velocity of a car over time
- Video signals
  - Intensity level of a pixel (camera, video) over time

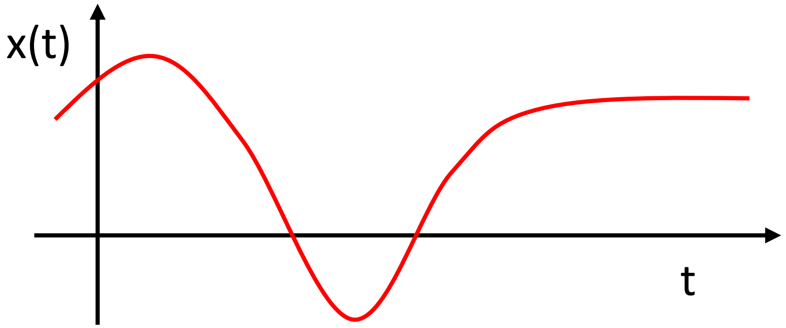
How is a Signal Represented?

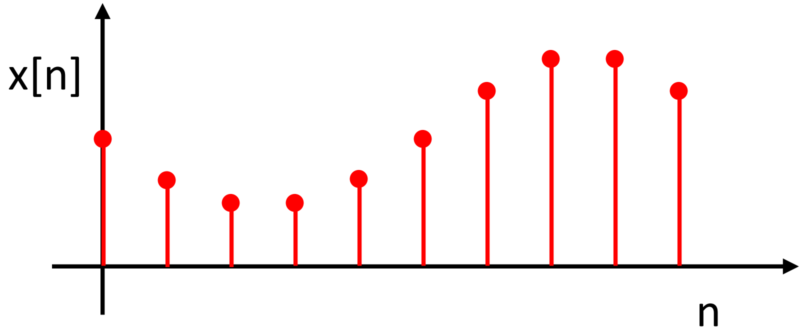
- Mathematically, signals are represented as a function of one or more independent variables.
- For instance a black & white video signal intensity is dependent on x, y coordinates and time t $f(x,y,t)$
- On this course, we shall be exclusively concerned with signals that are a function of a single variable: time



Continuous & Discrete-Time Signals

- **Continuous-Time Signals**
- Most signals in the real world are continuous time, as the scale is infinitesimally fine.
- Eg voltage, velocity,
- Denote by $x(t)$, where the time interval may be bounded (finite) or infinite
- **Discrete-Time Signals**
- Some real world and many digital signals are discrete time, as they are sampled
- E.g. pixels, daily stock price (anything that a digital computer processes)
- Denote by $x[n]$, where n is an integer value that varies discretely
- **Sampled continuous signal** $x[n] = x(nk)$ – k is sample time





Signal Properties

- On this course, we shall be particularly interested in signals with certain properties:
- **Periodic signals**: a signal is periodic if it repeats itself after a fixed period T, i.e. $x(t) = x(t+T)$ for all t. A sin(t) signal is periodic.
- **Even and odd signals**: a signal is even if $x(-t) = x(t)$ (i.e. it can be reflected in the axis at zero). A signal is odd if $x(-t) = -x(t)$. Examples are cos(t) and sin(t) signals, respectively.
- **Exponential and sinusoidal signals**: a signal is (real) exponential if it can be represented as $x(t) = Ce^{at}$. A signal is (complex) exponential if it can be represented in the same form but C and a are complex numbers.
- **Step and pulse signals**: A pulse signal is one which is nearly completely zero, apart from a short spike, $d(t)$. A step signal is zero up to a certain time, and then a constant value after that time, $u(t)$.

- These properties define a large class of tractable, useful signals and will be further considered in the coming lectures

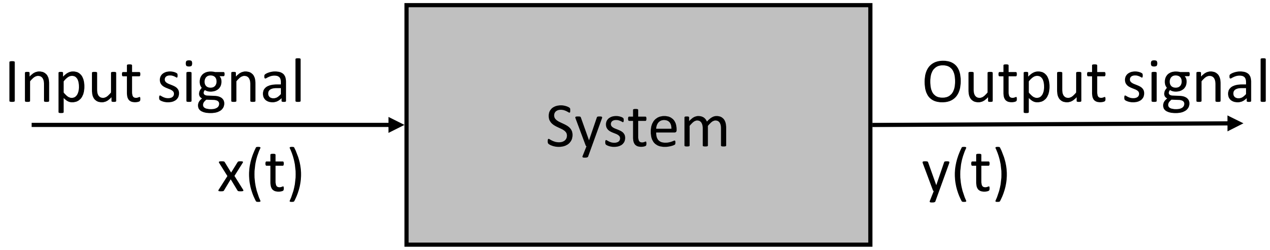
What is a System?

- Systems process input signals to produce output signals

- Examples:
  - A circuit involving a capacitor can be viewed as a system that transforms the source voltage (signal) to the voltage (signal) across the capacitor
  - A CD player takes the signal on the CD and transforms it into a signal sent to the loud speaker
  - A communication system is generally composed of three sub-systems, the transmitter, the channel and the receiver. The channel typically attenuates and adds noise to the transmitted signal which must be processed by the receiver

How is a System Represented?

- A system takes a signal as an input and transforms it into another signal



- In a very broad sense, a system can be represented as the ratio of the output signal over the input signal
  - That way, when we “multiply” the system by the input signal, we get the output signal
  - This concept will be firmed up in the coming weeks

Properties of a System

- On this course, we shall be particularly interested in signals with certain properties:
- **Causal**: a system is causal if the output at a time, only depends on input values up to that time.
- **Linear**: a system is linear if the output of the scaled sum of two input signals is the equivalent scaled sum of outputs
- **Time-invariance**: a system is time invariant if the system’s output is the same, given the same input signal, regardless of time.

- These properties define a large class of tractable, useful systems and will be further considered in the coming lectures

- 2. **Even and odd signals:**

Even:

$$x(-t) = x(t)$$

$$x[-n] = x[n]$$

Odd:

$$x(-t) = -x(t)$$

$$x[-n] = -x[n]$$

- Any signal x(t) can be expressed as

$$x(t) = \; xe(t) + xo(t) \,)$$

$$x(-t) = xe(t) - xo(t)$$

where

$$xe(t) = 1/2(x(t) + x(-t))$$

$$xo(t) = 1/2(x(t) - x(-t))$$

• 3. **Periodic and non-periodic signals:**

- **CT signal:** if $x(t) = x(t + T)$, then $x(t)$ is periodic.
- ❖ Smallest T=Fundamental period: To
- ❖ Fundamental frequency fo = 1/To (Hz or cycles/second)
- ❖ Angular frequency: $\omega_o = 2\pi/T_o$ (rad/seconds)

- **DT signal:** if $x[n] = x[n + N]$, then $x[n]$ is periodic.
- ❖ min(No): fundamental period
- ❖ Fo = 1/No (cycles/sample)
- ❖ $\Omega = 2\pi/N$ (rads/sample). If the unit of n is designated as dimensionless,
- ❖ then is simply in radians.

- **Note:** A sampled CT periodic signal may not be DT periodic.

  Any Condition addition of two periodic CT signals, resultant must be periodic signal ?

- 4. **Deterministic and random signals.**
- Deterministic signal: No uncertainty with respect to its value at any time
- Completely specified at any time

- Random signal: Uncertain before it occurs. E.g., thermal noise.

- **Energy and power signals:**

- CT signal x(t):

❖ Energy: E = $\int_{-\infty}^{\infty} x^2(t)dt$

❖ Power: P = $\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x^2(t)dt$

- DT signal x[n]:
- Energy: E = $\sum_{-\infty}^{\infty} x^2[n]$
- Power:

$$\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2[n]$$

- Energy signal: if 0 < E < ∞
- Power signal: if 0 < P < ∞

- Analog Signal and Digital Signal

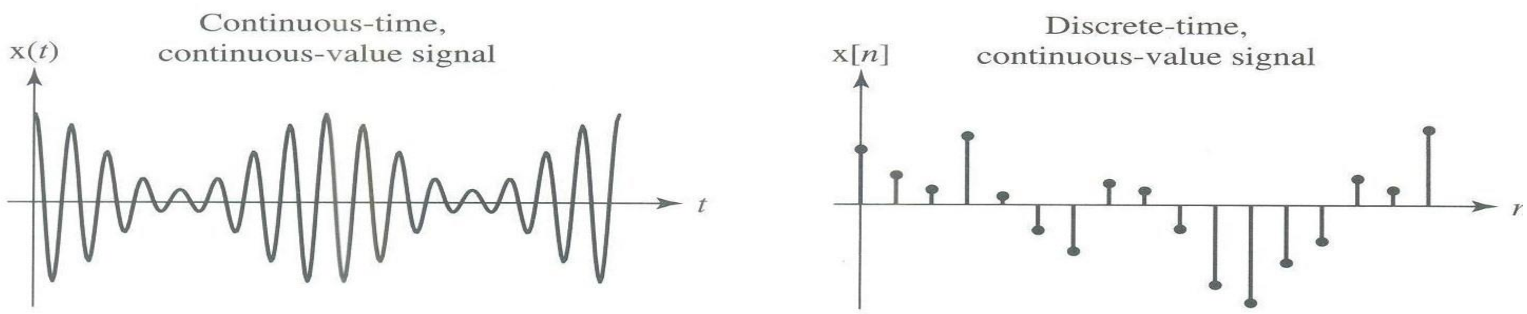


Figure
Examples of continuous-time and discrete-time signals.

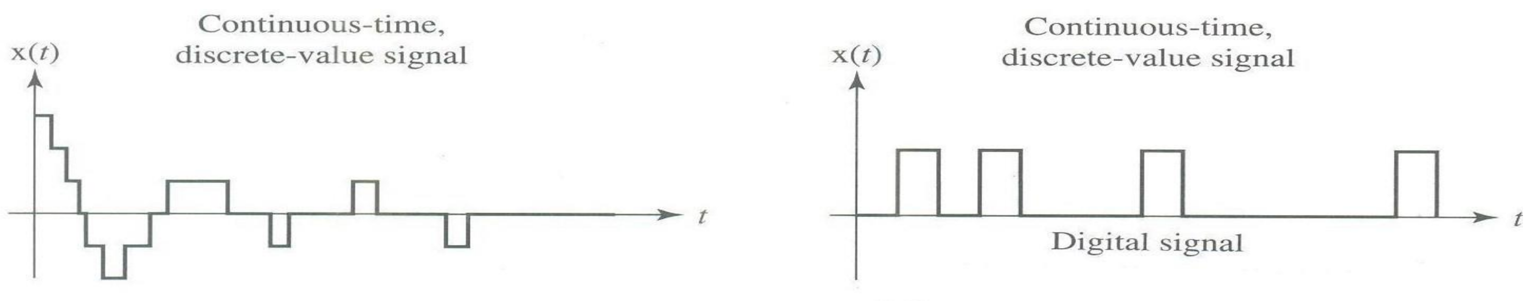


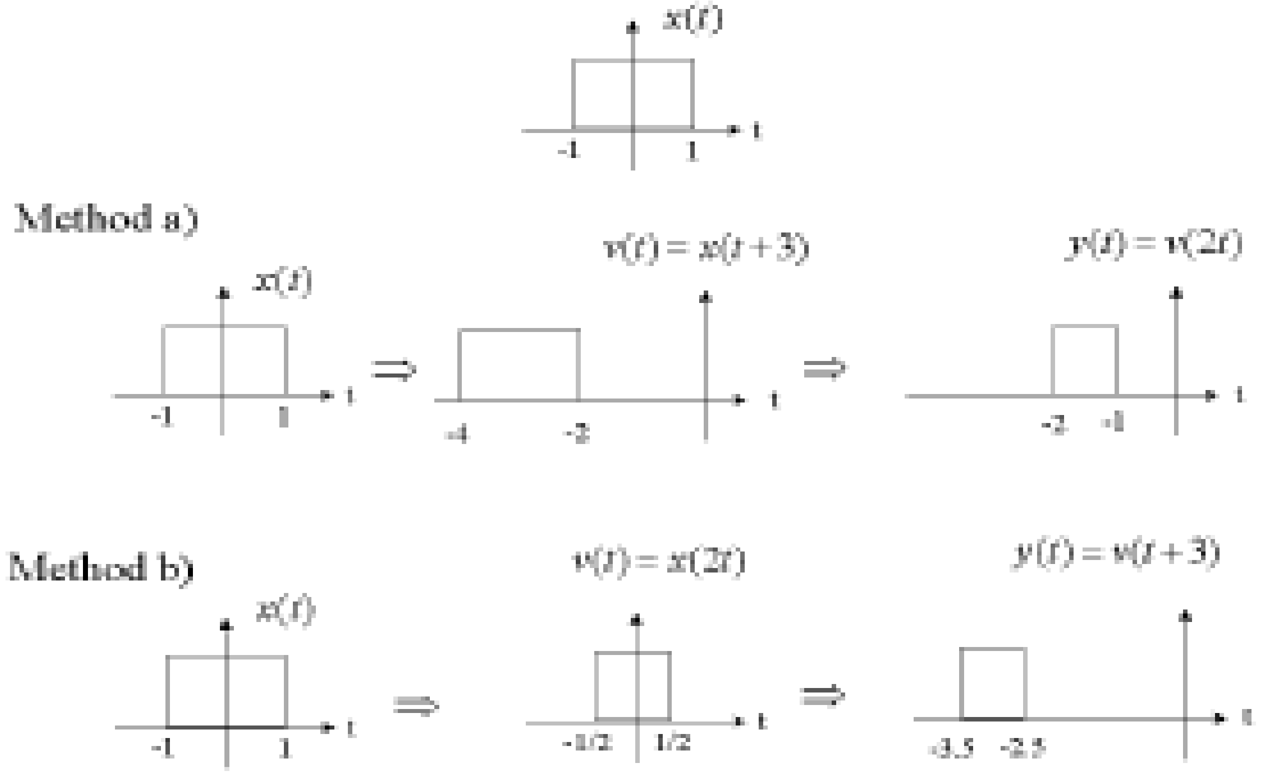
Figure
Examples of continuous-time and digital signals.

Operation	CT	DT	Note
Amplitude scaling	$y(t) = cx(t)$	$y[n] = cx[n]$	$c > 1$: gain $c < 1$: atten
Addition	$y(t) = x_1(t) + x_2(t)$	$y[n] = x_1[n] + x_2[n]$	
Multiplication	$y(t) = x_1(t)x_2(t)$	$y[n] = x_1[n]x_2[n]$	
Differentiation	$y(t) = \frac{d}{dt}x(t)$	(NO DT case)	
Integration	$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$	(NO DT case)	
Time scaling	$y(t) = x(at)$ $\begin{cases} a > 1 : & \text{compression} \\ a < 1 : & \text{expansion} \end{cases}$	$y[n] = x[kn]$ $k > 0$ and integer only	
Reflection (time reversal)	$y(t) = x(-t)$	$y[n] = x[-n]$	
Time shifting	$y(t) = x(t - t_0)$ $\begin{cases} t_0 > 0 : & \text{right shift} \\ t_0 < 0 : & \text{left shift} \end{cases}$	$y[n] = x[n - n_0]$ $\begin{cases} n_0 > 0 : & \text{right shift} \\ n_0 < 0 : & \text{left shift} \end{cases}$	
Combination	$y(t) = x(at - t_0)$	$y[n] = x[kn - n_0]$	

Basic Operations on Signal

- # Rule for time shifting and time scaling:

❖ See figure below. Find y(t) = x(2t + 3).



Basic Operations on Signal(cont.)

1. Exponential

CT	DT
$x(t) = Be^{at}$, a, B real	$x[n] = Br^n$
$\begin{cases} a < 0: & \text{decaying} \\ a > 0: & \text{growing} \\ a = 0: & \text{DC} \end{cases}$	$\begin{cases} 0 < r < 1: & \text{decaying} \\ r > 1: & \text{growing} \\ r = 1: & \text{DC} \end{cases}$

2-Sinusoidal

CT	DT
$x(t) = A\cos(\omega t + \phi)$	$x[n] = A\cos(\Omega n + \phi)$

Elementary signals

3. Step function

CT	DT
$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$	$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

Elementary signals(cont.)

4.Unit impulse function

CT	DT
$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t)dt = \int_{0-}^{0+} \delta(t)dt = 1 \end{cases}$	$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

5.Unit ramp function

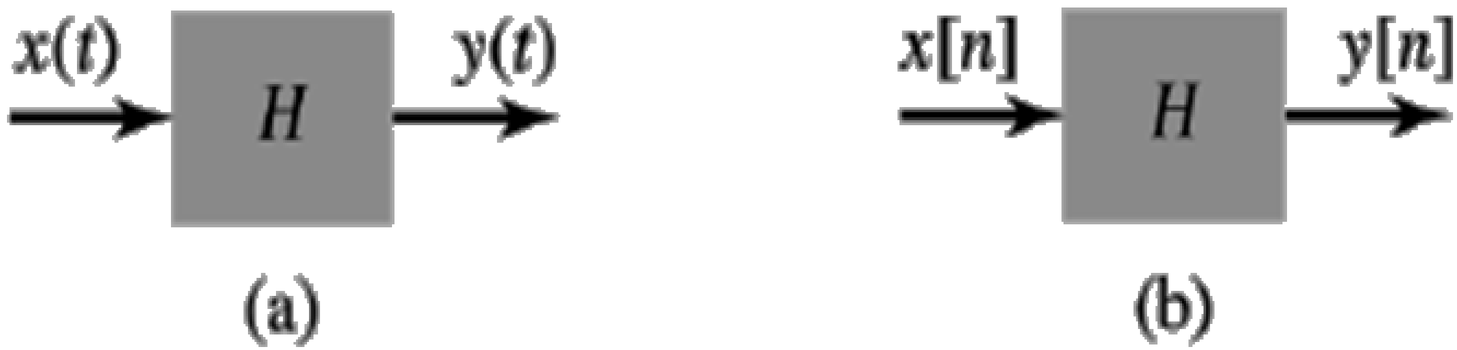
$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$r(t) = tu(t)$$

$$u(t) = \frac{d}{dt} r(t) = \frac{d}{dt} t = 1$$

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Elementary signals(cont.)



Notation: Let $\mathcal{H}$ represent the system, $x(t) \xrightarrow{\mathcal{H}} y(t)$ represent a system with input $x(t)$ and output $y(t)$.

1. Stability: BIBO (bounded input $\Rightarrow$ bounded output) stability

$$|x(t)| \leq M_x < \infty \implies |y(t)| \leq M_y < \infty$$
$$|x[n]| \leq M'_x < \infty \implies |y[n]| \leq M'_y < \infty$$

System Properties

2.Memory /Memoryless

- Memory system: present output value depend on future/past input.
- Memoryless system: present output value depend only on present input.
- Example

Memory systems:

$$y(t) = 5x(t) + \int_{-\infty}^{t} x(\tau)d\tau$$

$$y[n] = \sum_{m=n-5}^{n+5} x[m]$$

Memoryless systems:

$$y[n] = x[n] + x^2[n]$$

System Properties(cont.)

3. Causal/noncausal

- Causal: present output depends on present/past values of input.
- Noncausal: present output depends on future values of input.

Note: Memoryless ⇒ causal, but causal not necessarily be memoryless.

4. Time invariance (TI): time delay or advance of input ⇒ an identical time shift in the output.
Let us define a system mapping $y(t) = \mathcal{H}(x(t))$. The system is time-invariant if

$$x(t-t_0) \xrightarrow{\mathcal{H}} y(t-t_0)$$

$$x[n-n_0] \xrightarrow{\mathcal{H}} y[n-n_0]$$

System Properties(cont.)

5. Linearity

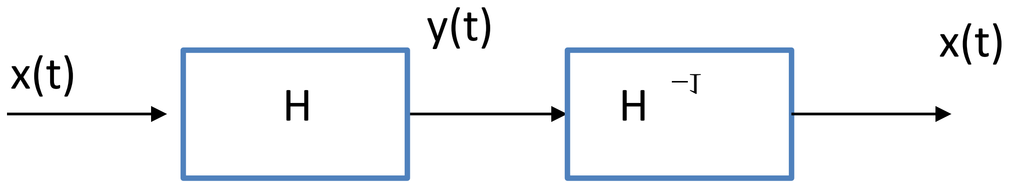
Linear system: If $x_1(t) \xrightarrow{\mathcal{H}} y_1(t)$, $x_2(t) \xrightarrow{\mathcal{H}} y_2(t)$, then $ax_1(t) + bx_2(t) \xrightarrow{\mathcal{H}} ay_1(t) + by_2(t)$. Else, nonlinear.

- Superposition property (addition)
- Homogeneity (scaling)

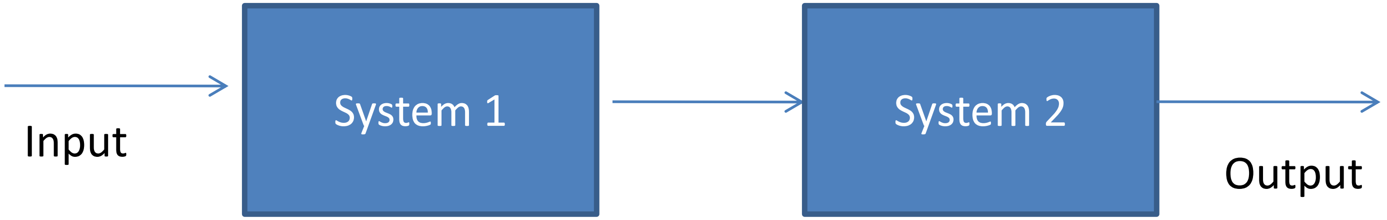
System Properties(cont.)

System Properties(cont.)

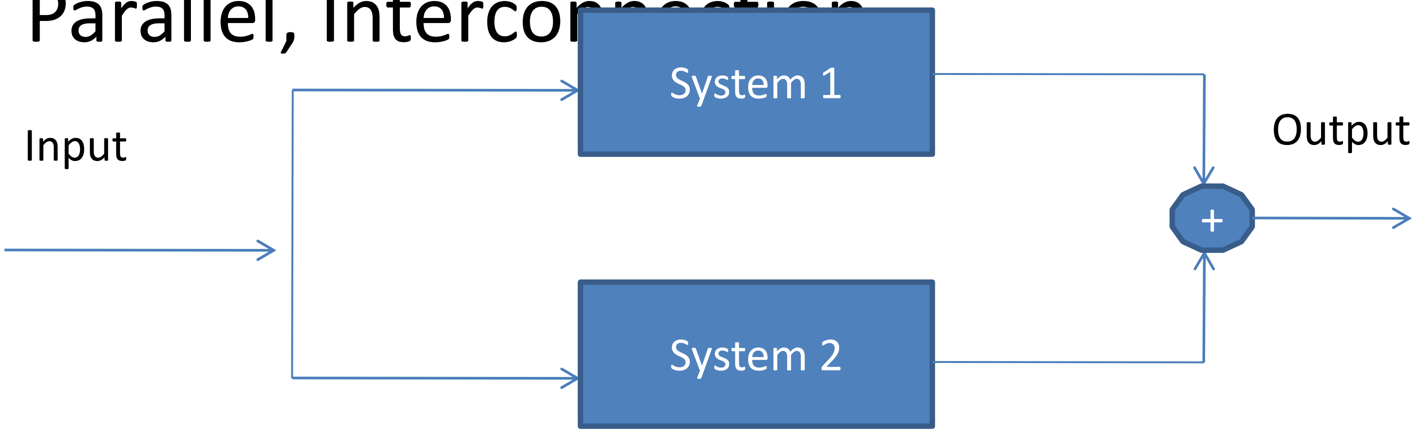
- Invertibility



- Series(cascade) Interconnection

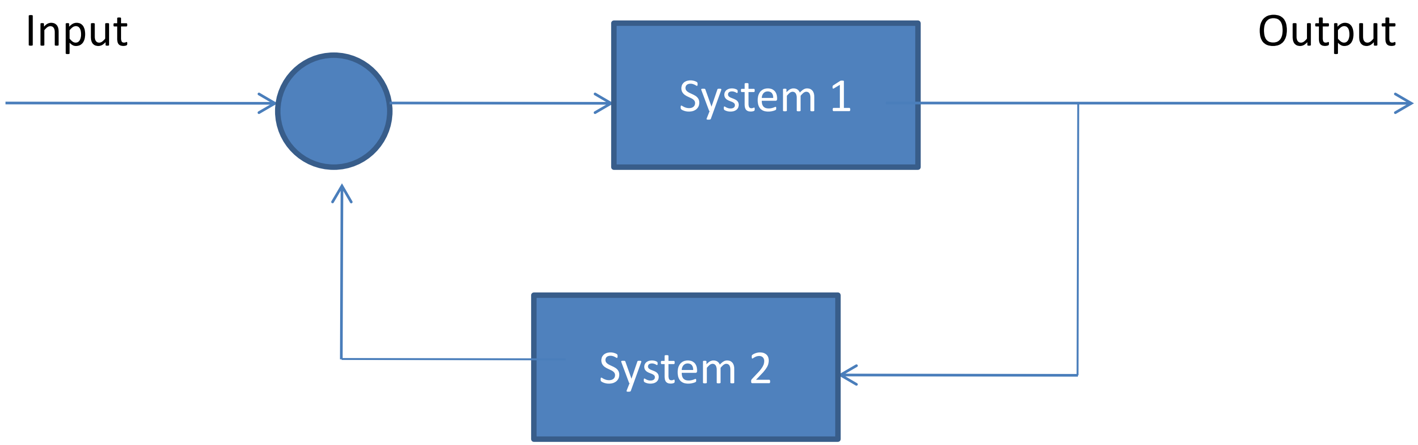


- Parallel, Interconnection



Interconnection of systems

- Feedback Interconnection



Interconnection of systems